Math 54 - Solutions to graded problems in Homework 4

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3.1.9

$$\begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix} = 2 \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{vmatrix} = 2 \times 5 \times \begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix} = 2 \times 5 \times (7-6) = 10$$

Where we expanded along the third row and the first row respectively.

4.2.13

$$W = \left\{ \begin{bmatrix} c \\ 0 \\ c \end{bmatrix} + \begin{bmatrix} -6d \\ d \\ 0 \end{bmatrix} \right\} = \left\{ c \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + d \begin{bmatrix} -6 \\ 1 \\ 0 \end{bmatrix} \right\} = Span \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Since the Span of any set of vectors is a vector space (Theorem 1), W is a vector space.

Alternatively: Notice that $W = Col(A)$, where $A = \begin{bmatrix} 1 & -6 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$. Since $Col(A)$ is a vector space (Theorem 3), W is a vector space.

4.3.33

Suppose $a\mathbf{p_1} + b\mathbf{p_2} = \mathbf{0}$, then:

$$\begin{aligned} a(1+t^2) + b(1-t^2) &= 0 \\ a + at^2 + b - bt^2 &= 0 \\ (a-b)t^2 + (a+b) &= 0 \end{aligned}$$

However, a polynomial equals to 0 if and only if its coefficients equal to 0. Hence we get $a - b = 0$ and $a + b = 0$, which implies $a = b = 0$. Hence $\mathbf{p_1}$ and $\mathbf{p_2}$ are linearly independent.

Alternative method:

$\mathbf{p_1}$ and $\mathbf{p_2}$ are linearly independent because $\mathbf{p_1}$ is not a multiple of $\mathbf{p_2}$ (and neither of them is the zero-polynomial).
WARNING: This trick only works if you have two polynomials!

Awesome Peyam method:

Let $\mathcal{B} = \left\{1, t, t^2, t^3\right\}$ be a basis for $\mathbb{P}_3$. Then $\mathbf{p_1}$ and $\mathbf{p_2}$ have coordinates $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ respectively. Since those two vectors are linearly independent in $\mathbb{R}^4$, $\mathbf{p_1}$ and $\mathbf{p_2}$ are linearly independent in $\mathbb{P}_3$. Oh the joy of coordinates!